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CBSE GRADE XII

BOARD EXAM

MATHEMATICS

RAPID
REVISION

**DIFFERENTIAL EQUATIONS,
PROBABILITY**

09-03-2023

THURSDAY- 04.00 P.M

DON'T MISS!



PROBABILITY



Q1

Consider the experiment of tossing a coin. If the coin shows head, toss it again, but if it shows tail, then throw a die. Find the conditional probability of the event that "the die shows a number greater than 4" given that "there is at least one tail"

$$S = \{(H, H), \underline{(H, T)}, \underline{(T, 1)}, \underline{(T, 2)}, \underline{(T, 3)}, \underline{(T, 4)}, \underline{(T, 5)}, \underline{(T, 6)}\}$$

A: die shows a number greater than 4 $(A \cap B) = \{(T, 5), (T, 6)\}$

B: at least one tail

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right)}{\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) \times 6}$$

$$= \frac{2}{9}$$

$$B = \{7 \text{ outcomes}\}$$

Q2

A problem in mathematics is given to three students whose chances of solving it are, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. What is the probability that:

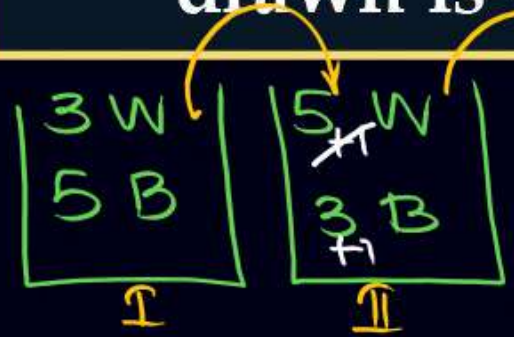
- (a) Exactly one of them solve the problem
- (b) Exactly two of them solve the problem $ABC' + AB'C + A'BC$
- (c) All three solve the problem $P(ABC) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$
- (d) None of them solve the problem $P(A'B'C') = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$
- (e) Problem is solved $= 1 - P(\text{none solves}) = 1 - \frac{1}{4} = \frac{3}{4}$

★★★
★★★

$$\begin{aligned}
 &P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C) \\
 &= \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{4}\right) \\
 &= \frac{6+3+2}{24} = \frac{11}{24}
 \end{aligned}$$

Q3

A bag contains 3 white and 5 black balls, a second bag contains 5 white and 3 black balls. One ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is white.



E_1 : first ball transferred is white
 E_2 : " " " " black
 F : ball is white from second bag

$$P(E_1) = \frac{3}{8}$$

$$P(E_2) = \frac{5}{8}$$

$$P(F|E_1) = \frac{6}{9}$$

$$P(F|E_2) = \frac{5}{9}$$

$P(\text{white ball from bag II})$

$$P(F) = P(E_1) \times P(F|E_1) + P(E_2) \times P(F|E_2)$$

$$= \frac{3}{8} \times \frac{6}{9} + \frac{5}{8} \times \frac{5}{9}$$

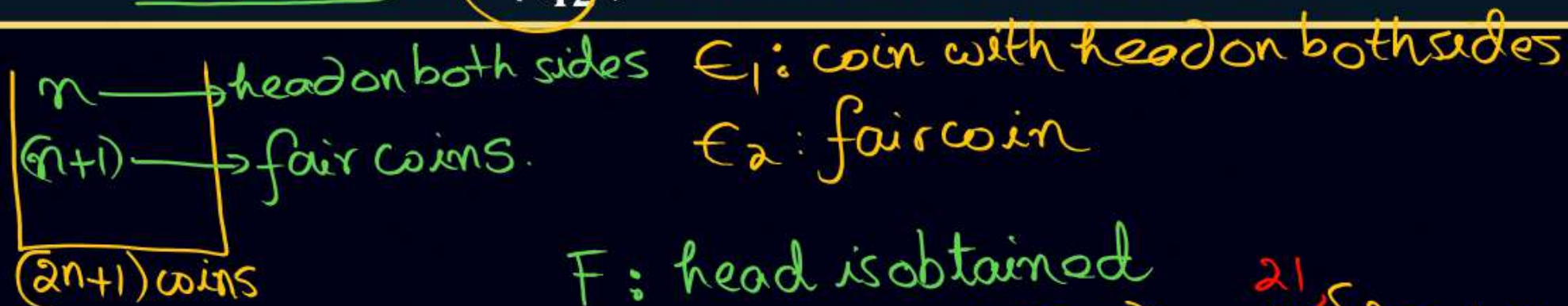
$$= \frac{18 + 25}{72}$$

$$= \frac{43}{72}$$



Q4

A bag contains $(2n + 1)$ coins, it is known that (n) of these coins have a head on both sides, whereas the rest of the coins are fair. A coin is picked at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{31}{42}$, determine the value of n .



$$P(F) = \frac{31}{42} \rightarrow \text{total prob.}$$

$$\frac{31}{42} = P(E_1) \cdot P(F|E_1) + P(E_2) \cdot P(F|E_2)$$

$$\frac{31}{42} = \frac{n}{(2n+1)} \cdot \left[\frac{2}{2}\right] + \frac{(n+1)}{(2n+1)} \times \frac{1}{2}$$

$$2(2n+1)31 = 42\{2n+n+1\}$$

$$62n + 31 = 63n + 21$$

$$10 = n$$



Q5

In a factory which manufactures bolts machines A, B and C manufacture respectively 25%, 35% and 40% respectively of the bolts. Of their outputs 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the products and is found to be defective. What is the probability that it is manufactured by the machine B?

E_1 : bolt from A $P(E_1) = \frac{25}{100}$, $P(E_2) = \frac{35}{100}$; $P(E_3) = \frac{40}{100}$
 E_2 : " " B
 E_3 : " " C $P(F|E_1) = \frac{5}{100}$ $P(F|E_2) = \frac{4}{100}$ $P(F|E_3) = \frac{2}{100}$
F : defective bolt

(Bayes' thm) =

$$P(E_2|F) = \frac{P(E_2) \times P(F|E_2)}{P(E_1) \times P(F|E_1) + P(E_2) \times P(F|E_2) + P(E_3) \times P(F|E_3)}$$
$$= \frac{\cancel{25} \times 4}{\cancel{25} \times 5 + \cancel{35} \times 4 + \cancel{40} \times 2} = \frac{38}{69}$$



Q6

Suppose the reliability of a HIV test is specified as follows. Of the people having HIV, 90% of the test detects the disease but 10% go undetected. Of the people not having HIV, 99% of the test is judged HIV negative but 1% are diagnosed HIV positive. From a large population of which only 0.1% have HIV, one person is selected at random, and is given the HIV test, and the pathologist reports as HIV positive. What is the probability that the person actually has HIV?

E_1 : person selected has HIV
 E_2 : person selected has no HIV

F : report is positive

$$P(E_1) = \frac{0.1}{100} ; P(E_2) = \frac{99.9}{100}$$

$$P(F|E_1) = \frac{90}{100} ; P(F|E_2) = \frac{1}{100}$$

$$\begin{aligned} P(E_1|F) &= \frac{0.1 \times 90}{0.1 \times 90 + 99.9 \times 1} \\ &= \frac{9}{9 + 99.9} = \frac{9}{108.9} \\ &= \underline{\underline{0.083}} \end{aligned}$$



Q7

In a test an examinee either guesses or copies or knows the answer to the multiple choice questions with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

E_1 : guess the answer

E_2 : copies

E_3 : knows the answer

F : correct answer.

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{6}, P(E_3) = 1 - \left[\frac{1}{3} + \frac{1}{6} \right]$$

MCQ with 4 opt. = $\frac{1}{2}$

$$P(F|E_1) = \frac{1}{4}, P(F|E_2) = \frac{1}{8}, P(F|E_3) = 1$$

$$P(E_3|F) = \frac{\frac{1}{2} \times 1}{\left(\frac{1}{3} \times \frac{1}{4} \right) + \left(\frac{1}{6} \times \frac{1}{8} \right) + \left(\frac{1}{2} \times 1 \right)}$$
$$= \frac{1}{\frac{1}{6} + \frac{1}{24} + 1}$$
$$= \frac{24}{4 + 1 + 24} = \frac{24}{29}$$



Q8

Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a particular meditation / yoga course reduces the risk of heart attack by 30% and the prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation / yoga.



Q9

A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn at random and are found to be both diamonds. Find the probability of the lost card being a diamond.



Q10

Bag I contains 3 red and 4 black balls and bag II contains 4 red and 5 black balls. One ball is transferred from bag I to bag II and then a ball is drawn from bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.



Q11

Find the probability distribution of number of tails in the simultaneous tosses of four coins. Also find the mean

$$\text{mean} = \sum (x \cdot p) = \underline{2}$$

X : number of tails in 4 tosses. ${}^4C_1 = \frac{4}{1}$ ${}^4C_3 = \frac{4 \times 3 \times 2}{3 \times 2 \times 1} = 4$
 i.e. $X = 0, 1, 2, 3, 4$ ${}^nC_n = {}^nC_0 = 1$ ${}^4C_2 = \frac{4 \times 3}{2 \times 1}$

$$P(X=0) \Rightarrow (HHHH) \Rightarrow \left\{ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right\} \times {}^4C_0 = \frac{1}{16} \times 1$$

$$P(X=1) \Rightarrow (T H H H) \Rightarrow \left\{ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right\} \times {}^4C_1 = \frac{1}{16} \times 4$$

$$P(X=2) \Rightarrow (T T H H) \Rightarrow \left\{ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right\} \times {}^4C_2 = \frac{1}{16} \times 6$$

$$P(X=3) \Rightarrow (T T T H) \Rightarrow \left\{ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right\} \times {}^4C_3 = \frac{1}{16} \times 4$$

$$P(X=4) \Rightarrow (T T T T) \Rightarrow \left\{ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right\} \times {}^4C_4 = \frac{1}{16} \times 1$$

X	$P(X)$	$X \cdot P$
0	$\frac{1}{16}$	0
1	$\frac{4}{16}$	$\frac{4}{16}$
2	$\frac{6}{16}$	$\frac{12}{16}$
3	$\frac{4}{16}$	$\frac{12}{16}$
4	$\frac{1}{16}$	$\frac{4}{16}$

$$\underbrace{1}_{\sum} \quad \underbrace{\frac{32}{16}}_{\sum} = 2 //$$



Q12

Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Find the mean of the number of aces obtained.



X : number of aces obtained in the 2 cards with replacement.

$$\text{mean} = \sum (x \cdot p) \\ = \frac{26}{169} = \frac{2}{13}$$

$X: 0, 1, 2$

$$P(X=0) = \left[\frac{48}{52} \times \frac{48}{52} \right] \times {}^2C_0 = \frac{144}{169} \times 1 = \frac{144}{169}$$

$$P(X=1) = \left[\frac{4}{52} \times \frac{48}{52} \right] \times {}^2C_1 = \frac{12 \times 2}{169} = \frac{24}{169}$$

$$P(X=2) = \left[\frac{4}{52} \times \frac{4}{52} \right] \times {}^2C_2 = \frac{1}{169} = \frac{1}{169}$$

X	$P(X)$	$X \cdot P(X)$
0	$\frac{144}{169}$	0
1	$\frac{24}{169}$	$\frac{24}{169}$
2	$\frac{1}{169}$	$\frac{2}{169}$
	$\frac{1}{169}$	$\frac{26}{169}$



Q13

Two cards are drawn simultaneously from a well shuffled deck of 52 cards. Find the mean of the number of kings obtained.

without replacement

X : no. of kings obtained

$$X = 0, 1, 2$$

$$P(X=0) = \left\{ \frac{48}{52} \times \frac{47}{51} \right\} = \frac{4 \times 47}{13 \times 17} = \frac{188}{221}$$

$$P(X=1) = \left\{ \frac{4}{52} \times \frac{48}{51} \right\} \times {}^2C_1 = \frac{16 \times 2}{13 \times 17} = \frac{32}{221}$$

$$P(X=2) = \left\{ \frac{4}{52} \times \frac{3}{51} \right\} = \frac{1}{13 \times 17} = \frac{1}{221}$$

X	P	X · P
0	$\frac{188}{221}$	0
1	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$
	1	$\frac{34}{221}$

$$\text{mean} = \frac{34}{221} = \frac{2 \times 17}{13 \times 17} = \frac{2}{13}$$



Q14

Let a pair of dice be thrown and the random variable X be the sum of the numbers that appear on the two dice. Find the expectation of X .



Q15

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find mean.



1) variable separable forms

2) Homogeneous diff. eqn

3) Linear diff. eqn.

type 1: $\frac{dy}{dx} = \frac{F(x,y)}{G(x,y)}$

→ $\frac{y}{x}$ form

degree of every term is same

type 2: $\frac{dx}{dy} = \frac{F(x,y)}{G(x,y)}$

→ only if $\left[\frac{x}{y}\right]$ term is visible in the question

type 1

$$\frac{dy}{dx} + P(y) = Q$$

type 2

$$\frac{dx}{dy} + P(x) = S$$

DIFFERENTIAL EQUATIONS



Q1

Find the general solution of the differential equation $y dx - (x + 2y^2) dy = 0$

$$y dx - [x + 2y^2] dy = 0$$

$$y dx = (x + 2y^2) dy$$

$$y \frac{dx}{dy} = x + 2y^2$$

$$y \frac{dx}{dy} - x = 2y^2$$

$$\frac{dx}{dy} - \frac{1}{y} \cdot x = 2y$$

$$\rightarrow R = \frac{-1}{y}, S = 2y$$

$$1. \frac{dx}{dy} + Rx = S$$

$$\text{IF} = e^{\int R \cdot dy} = e^{\int -1/y \cdot dy} = e^{-\log y} = e^{\log y^{-1}} = y^{-1}$$

$$\therefore \text{IF} = \frac{1}{y}$$

$$\text{Soln } [x \cdot \text{IF}] = \int (S \cdot \text{IF})$$

$$x \cdot \frac{1}{y} = \int 2y \cdot \frac{1}{y} dy$$

$$\frac{x}{y} = 2y + C$$

$$x = 2y^2 + Cy$$

Q2

Solve the differential equation,

$$\frac{dy}{dx} + y \cdot \cot(x) = (2x + x^2 \cot x) y \quad y\left(\frac{\pi}{2}\right) = 0$$

$$\frac{dy}{dx} + P \cdot y = Q$$

$$\rightarrow \text{IF} = e^{\int P \cdot dx}$$

$$= e^{\int \cot x \cdot dx}$$

$$= e^{\log|\sin x|}$$

$$= |\sin x|$$

type 1 soln

$$(y \times \text{IF}) = \int (Q \times \text{IF})$$

$$y \cdot \sin x = \int (2x + x^2 \cot x) \cdot \sin x \cdot dx$$

$$y \cdot \sin x = \int 2x \cdot \sin x \cdot dx + \int x^2 \cdot \cot x \cdot \sin x \cdot dx$$

$$+ \int x^2 \cdot \cos x \cdot dx$$

$$+ \{x^2 \int \cos x - \int d(x^2) \cdot \int \cos x\}$$

$$= \int 2x \sin x \cdot dx + \{x^2 \sin x - \int 2x \sin x\}$$

$$y \sin x = x^2 \sin x + C \rightarrow \textcircled{1} y=0; x=\pi/2$$

$$0 = \frac{\pi^2}{4} + C$$

$$C = -\frac{\pi^2}{4}$$

$$\therefore \textcircled{1} \Rightarrow y \sin x = x^2 \sin x - \frac{\pi^2}{4}$$



Q3

Solve the differential equation,

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} \right) \cdot \frac{dx}{dy} = 1, x \neq 0$$

$$\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} = \frac{dy}{dx}$$

$$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{1}{\sqrt{x}} ; Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$\begin{aligned} \text{IF} &= e^{\int P \cdot dx} = e^{\int \frac{1}{\sqrt{x}} \cdot dx} \\ &= e^{2\sqrt{x}} \end{aligned}$$

type 1

$$y \cdot \text{IF} = \int Q \cdot \text{IF}$$

$$y \cdot e^{2\sqrt{x}} = \int \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \times e^{2\sqrt{x}}$$

$$y e^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}} \cdot dx$$

$$y e^{2\sqrt{x}} = 2\sqrt{x} + C$$



Q4

Solve the differential equation,

$$(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}; y = 0 \text{ when } x = 1$$

$$\frac{dy}{dx} + Py = Q$$

$$\frac{dy}{dx} + \frac{2x}{(1+x^2)} y = \frac{1}{(1+x^2)^2}$$

$$IF = e^{\int \frac{2x}{1+x^2}} \rightarrow \int \frac{f'}{f} = \log f$$

$$= e^{\log(1+x^2)}$$

$$= (1+x^2)$$





Q5

Solve the differential equation,

$$(\sin^{-1} y - x) dy = \sqrt{1 - y^2} dx$$

$$\frac{dx}{dy} + Rx = S$$

$$(\sin^{-1} y - x) = \sqrt{1 - y^2} \cdot \frac{dx}{dy}$$

$$\sqrt{1 - y^2} \cdot \frac{dx}{dy} + x = \sin^{-1} y$$

$$\frac{dx}{dy} + \frac{x}{\sqrt{1 - y^2}} = \frac{\sin^{-1} y}{\sqrt{1 - y^2}}$$

$$x \cdot e^{\int \frac{1}{\sqrt{1 - y^2}} dy} = \int \frac{\sin^{-1} y}{\sqrt{1 - y^2}} \cdot e^{\int \frac{1}{\sqrt{1 - y^2}} dy} dy + C$$

$$IF = e^{\int R dy} = e^{\int \frac{1}{\sqrt{1 - y^2}} dy} = e^{\sin^{-1} y}$$

$$\text{let } t = \sin^{-1} y \\ dt = \frac{1}{\sqrt{1 - y^2}} dy$$

Soln is:

$$x \cdot IF = \int S \cdot IF$$

$$R = \frac{1}{\sqrt{1 - y^2}} \\ S = \frac{\sin^{-1} y}{\sqrt{1 - y^2}}$$

$$x \cdot e^{\sin^{-1} y} = \int \frac{\sin^{-1} y}{\sqrt{1 - y^2}} e^{\sin^{-1} y} dy$$

$$= \int t \cdot e^t dt$$

$$= t \cdot e^t - \int 1 \cdot e^t \\ = t \cdot e^t - e^t + C$$

Q6

Solve the differential equation.

$$(x)dy - (y)dx = \sqrt{x^2 + y^2} \cdot dx$$

HDE; type 1 $\Rightarrow \frac{dy}{dx} = \frac{F}{G}$

$$x \cdot dy - y \cdot dx = \sqrt{x^2 + y^2} \cdot dx$$

$$x \cdot \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

$$x \cdot \frac{dy}{dx} = \sqrt{x^2 + y^2} + y$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$$

Let $y = vx \rightarrow v = \frac{y}{x}$

$$\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{\sqrt{x^2 + v^2 x^2} + vx}{x}$$

~~$$v + x \cdot \frac{dv}{dx} = \frac{x \sqrt{1 + v^2} + vx}{x}$$~~

$$x \cdot \frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x} \Rightarrow \log |v + \sqrt{1 + v^2}| = \log x + \log C$$

~~$$\log |v + \sqrt{1 + v^2}| = \log(Cx)$$~~

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = Cx$$

$$y + \sqrt{x^2 + y^2} = Cx^2$$



Q7

Show that the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

Show homogeneous.

$$x \cdot \cos\left(\frac{y}{x}\right) \cdot \frac{dy}{dx} = y \cdot \cos\left(\frac{y}{x}\right) + x$$

$$\frac{dy}{dx} = \frac{y \cdot \cos\left(\frac{y}{x}\right) + x}{x \cos\left(\frac{y}{x}\right)}$$

let $y = \lambda y$; $x = \lambda x$

$$\begin{aligned} \rightarrow \frac{dy}{dx} &= \frac{\lambda y \cdot \cos\left(\frac{y}{x}\right) + \lambda x}{\lambda x \cdot \cos\left(\frac{y}{x}\right)} \\ &= \frac{\lambda \{y \cdot \cos\left(\frac{y}{x}\right) + x\}}{\lambda x \cos\left(\frac{y}{x}\right)} \end{aligned}$$

$$\frac{dy}{dx} = \frac{y \cos\left(\frac{y}{x}\right) + x}{x \cos\left(\frac{y}{x}\right)}$$

HDE degree = 0

let $y = vx$; $\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$



Q8

Solve the differential equation,

$$2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2 \text{ when } x = 1$$

$$2x^2 \cdot \frac{dy}{dx} = 2xy + y^2$$

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

$$y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$$

$$v + x \cdot \frac{dv}{dx} = \frac{2x \cdot vx + v^2 x^2}{2x^2}$$

$$v + x \cdot \frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$\cancel{v} + x \cdot \frac{dv}{dx} = \cancel{v} + \frac{v^2}{2}$$

$$\int \frac{dv}{v^2} = \int \frac{1}{2} \frac{dx}{x}$$

$$\frac{-1}{v} = \frac{1}{2} \log x + C$$

$$\frac{-x}{y} = \frac{1}{2} \log x + C$$



Q9

Solve the differential equation,

$$y. \left(x. \cos \frac{y}{x} + y. \sin \frac{y}{x} \right) dx - x. \left(y. \sin \frac{y}{x} - x. \cos \frac{y}{x} \right). dy = 0$$



Q10

Solve the differential equation,

$$2y e^{\frac{x}{y}} dx + \left(y - 2x e^{\frac{x}{y}} \right) dy = 0, \text{ given that, } x = 0 \text{ when } y = 1.$$



Q11

Find the general solution of the differential

equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$



Q12

Solve the differential equation,

$$(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$$



Q13

Solve the differential equation,

$$\sqrt{1 + x^2 + y^2 + x^2 y^2} + xy \frac{dy}{dx} = 0$$